$Y_1^{*+} - Y_1^{*-}$ Mass Difference

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The $Y_1^{*+} - Y_1^{*-}$ mass difference is calculated, taking account of the contributions from (1) scalar-meson tadpoles, (2) intermediate states involving one Y_1^* and one photon, and (3) intermediate states involving one Σ and one photon. The sum of these yields a mass splitting of between -8.5 and -10.8 MeV. The experimental value is -17 ± 7 MeV.

RECENTLY, it has been proposed[1] that the dominant contribution to both the medium-strong and the electromagnetic mass splittings of the strongly interacting particles comes from a class of diagrams involving scalar meson tadpoles. [Figure 1(a) shows such a diagram.] The medium-strong splittings come from diagrams involving a hypothetical isospin zero, hypercharge zero meson η'. The electromagnetic splittings come from the neutral member of a hypothetical hypercharge zero isotriplet π_0'. Because these particles are part of the same unitary octet, we may use unitary symmetry to obtain an estimate of the electromagnetic mass splittings in terms of the medium strong ones (to within a multiplicative constant). We may improve this estimate by adding to it the contributions of the leading nontadpole diagrams, those which involve in the usual dispersive analysis of the electromagnetic mass problem, the intermediate states of lowest mass.

This method is capable in principle of calculating all electromagnetic splittings in terms of experimental form factors and one unknown parameter. It has been applied[2] to the baryon and pseudoscalar meson octets. Good agreement is obtained with the baryon splittings and with the pion splittings. Poorer, but still reasonable, agreement (the right sign but only half the right magnitude) is obtained with the kaon splitting. These results are encouraging, but they are only half a dozen. We feel it is important to check the tadpole dominance hypothesis by applying the method to as many electromagnetic splittings as possible. In this note we apply it to the computation of the recently measured[3] mass difference between the charged states of the Y_1^*, the $\frac{3}{2}^+$ $\Lambda\pi$ resonance at 1385 MeV.

In our computation of the difference between the mass of Y_1^{*+} and that of Y_1^{*-}, we include the contributions from three mechanisms: (1) Scalar meson tadpoles [Fig. 1(a)]. The magnitude of this contribution can be calculated with no free parameters if we accept the analysis of baryon and pseudoscalar meson mass splittings in terms of tadpole dominance.[1] It is -6.1 MeV. (2) The Y_1^* pole [Fig. 1(b)]. As a consequence of unitary symmetry this has no effect on the difference. (3) The Σ pole [Fig. 1(c)]. This involves the unknown $\Sigma Y_1^* \gamma$ form factor. Unitary symmetry tells us the value of this form factor at zero-momentum transfer in terms of the known $NN^*\gamma$ coupling. We assume its shape is given by a single pole. As the mass of this pole moves from the mass of the ρ to the mass of the ϕ, this contribution moves from -2.4 to -4.7 MeV; we use these as lower and upper estimates.

The sum of these three terms yields a mass splitting of between -8.5 and -10.8 MeV.[4] The experimental value[3] is -17 ± 7 MeV.

The remainder of this note is a more detailed explanation of these calculations.

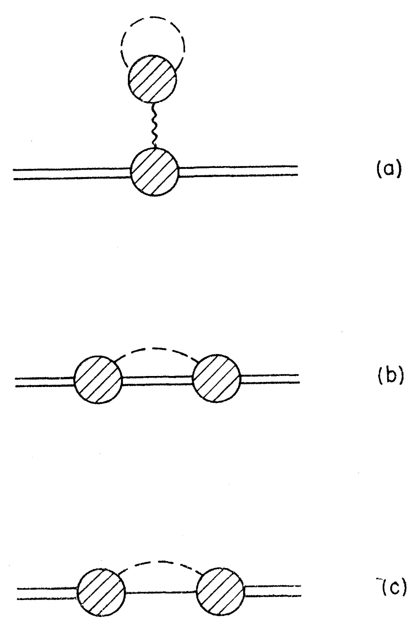

FIG. 1. The three leading contributions to the electromagnetic mass splittings within the decuplet. The dashed lines represent photons, the wiggly lines scalar mesons, the straight lines baryons, and the double lines decuplet resonances. These diagrams are obtained by writing the expression for the decuplet self-energy in terms of the scattering of unphysical photons off decuplets; the latter process is then approximated by including all poles in s, t, and u. Thus the blobs represent form factors with all particles except the photon on the mass shell.

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[1] S. Coleman and S. L. Glashow, Phys. Rev. **134**, B671 (1964).

[2] S. Coleman and H. Schnitzer (to be published).

[3] W. A. Cooper, H. Filthuth, A. Fridman, E. Malamud, E. S. Gelsema, J. C. Kluyver, A. G. Tenner, Phys. Letters **8**, 365 (1964).

[4] We may apply the same methods to a calculation of the $N^{*+} - N^{*-}$ splitting; we obtain the same splitting to within a few tenths of an MeV.

(*1*) *Scalar meson tadpoles.* Let g_D be the coupling of the scalar meson octet to the decuplet. Then the medium-strong mass splitting between, say, the Y_1^* and the N^* (1238 MeV), is given by

$$Y_1^* - N^* = -g_D\sqrt{3}\langle\eta'\rangle\,, \tag{1}$$

while the desired electromagnetic splitting is given by

$$Y_1^{*+} - Y_1^{*-} = 4g_D\langle\pi^{0\prime}\rangle\,, \tag{2}$$

where the name of a particle denotes its mass, and $\langle\eta'\rangle$ and $\langle\pi^{0\prime}\rangle$ are the vacuum expectation values of the associated scalar fields. According to the fit to the baryon masses of Coleman and Schnitzer[2], $\langle\pi'\rangle/\langle\eta'\rangle = 0.018$. Thus, the contribution of tadpoles to the $Y_1^{*+} - Y_1^{*-}$ splitting is -6.1 MeV.

(*2*) *The Y_1^* pole.* There is only one way of coupling 8 to $10\otimes\overline{10}$; thus all electromagnetic form factors for the decuplet are proportional to the charge. Hence the diagram of the type of Fig. 1(b) for the Y_1^{*+} is equal in magnitude to that of the same type for the Y_1^{*-}, and there is no effect on the difference. This is a fortunate cancellation: If we approximate the Y_1^* by a Rarita-Schwinger[5] field of spin $\frac{3}{2}$, the diagrams under consideration are at best quadratically divergent; their values depend sensitively on the way in which the Y_1^* becomes soft at high energies, and their proper calculation requires a detailed theory of the dynamics of the decuplet resonances. Further, even if we are willing to assume a one pole form for each of the form factors, we only know the values of one of them (the charge form factor) at zero momentum transfer; we know nothing of the various anomalous moments except that, judging by the example of the nucleon, they are likely to be large.

(*3*) *The Σ pole.* Gourdin and Salin[6] are able to describe many phenomena involving the electromagnetic interactions of the N^* with a phenomenological interaction Lagrange density of the form

$$\mathcal{L}' = (fe/\mu)[\bar{\Psi}\gamma_\mu\gamma_5\Psi_\nu + \bar{\Psi}_\nu\gamma_5\gamma_\mu\Psi]F^{\mu\nu}\,, \tag{3}$$

where Ψ is the nucleon field, Ψ_μ is a Rarita-Schwinger[5] field of spin $\frac{3}{2}$ associated with the N^*, $F^{\mu\nu}$ is the electromagnetic field, μ is the mass of the pion, and f is 0.37.[7] Unitary symmetry tells us that the $Y_1^*\Sigma^+\gamma$ coupling is given by an interaction of the same form with the same coupling constant, and that the $Y_1^{*-}\Sigma^-\gamma$ coupling vanishes.[8] In order to evaluate Fig. 1(c), we need to know the form factor that describes the coupling for photons off the mass shell; we will assume it is given by a single pole:

$$F(k^2) = fM^2(k^2-M^2)^{-1}\,. \tag{4}$$

A lengthy but straightforward Feynman calculation[9] yields the following expression for the electromagnetic mass of the Y_1^{*+} (and therefore for the splitting)

$$\delta m = \frac{m^3f^2e^2}{192\pi^2\mu^2}\Bigg[3x^3 - 5x^2y + xy^2 - \frac{7}{2}x^2 + 4xy + GF + H\ln\frac{1+y}{x} + (y^4+4y^3)\ln\frac{1+y}{|y|}\Bigg]\,, \tag{5}$$

where

$$x = M^2 m_{Y^*}^{-2}\,,\quad y = (m_\Sigma^2 - m_{Y^*}^2)m_{Y^*}^{-2}\,,$$

$$F = \int_0^1 (\alpha^2 + y\alpha - x\alpha + x)^{-1}d\alpha\,,$$

$$G = \tfrac{1}{2}(x-y)^4(y+3x) + (x-y)^2(2y^2+xy-7x^2) + 8x^3 - 8x^2y - 4y^2x - 4x^2 - 6(y+1)^{1/2}x^2(2+y-x)\,,$$

and

$$H = \tfrac{1}{2}(x-y)^3(3x+y) - 2(x-y)^2(2x+y) + 3x^2 + 6(1+y)^{1/2}x^2\,.$$

If we place M as the mass of the ρ, δm is -2.4 MeV; if M is at the mass of the ϕ, δm is -4.7 MeV. We use these numbers as lower and upper estimates. [Since we have no experimental or theoretical information about $F(k^2)$, this is at best just an educated guess. However, we remark that if we applied the same procedure to the p–n difference, we would obtain estimates such that the number obtained from best fits to the experimental form factors would indeed lie between them.]

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[5] W. Rarita and J. Schwinger, Phys. Rev. **60**, 61 (1941).

[6] M. Gourdin and Ph. Salin, Nuovo Cimento **27**, 193 (1963); **27**, 309 (1963); Ph. Salin, Nuovo Cimento **28**, 1294 (1963).

[7] There are two other possible gauge-invariant interactions. The coefficient of one of them is experimentally zero. The other automatically vanishes for photons on the mass shell; thus we learn nothing about its coefficient from experiments involving photoproduction or low momentum-transfer electroproduction. Of course, we cannot exclude the possibility that this constant is in fact large, and that this interaction makes a considerable contribution to the electromagnetic masses of the resonances.

[8] The simplest way to show this is through arguments based on U spin [C. Levinson, H. Lipkin, and S. Meshkov, Phys. Letters **1**, 44 (1962)]. The photon is a U-spin singlet. The positively charged resonances and the positively charged baryons are both U-spin doublets, and therefore couple electromagnetically with a single coupling constant. The negatively charged resonances are a U-spin quadruplet, while the negatively charged baryons are a U-spin doublet; therefore these can not couple electromagnetically at all.

[9] R. Socolow, thesis, Harvard University, 1964 (unpublished).